Collapse of the Fractional Quantum Hall Effect in an Electron System with Large Layer Thickness

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We report the first experimental study of the energy gap for the fractional quantum Hall states at Landau-level filling factors \( \nu = \frac{1}{3} \) and \( \frac{2}{3} \) in a variable-width electron system realized in a selectively doped, parabolic, AlGaAs quantum well. We observe a dramatic decrease in the measured energy gaps as the electron-layer thickness increases and the electric-subband separation decreases, which we interpret as the collapse of the correlated fractional states.

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The fractional quantum Hall effect (FQHE),\(^1\) observed in a two-dimensional electron system (2DES) in the presence of a strong perpendicular magnetic field \( (B) \), is the manifestation of a series of incompressible electron liquid states with strong short-range correlation but no long-range order.\(^2\) A fundamental characteristic of the FQHE is the existence of an energy gap \( (\Delta) \) separating the ground state from its elementary quasiparticle and quasihole excitations. There have been a number of recent experimental and theoretical investigations of the size of \( \Delta \).\(^3-11\) For the FQHE excitations at the Landau-level filling factor \( \nu = \frac{1}{3} \), the calculations for an ideal 2DES indicate \( \Delta = 0.1e^2/\epsilon l_0 \), where \( \epsilon \) is the dielectric constant and \( l_0 = (\hbar/eB)^{1/2} \) is the magnetic length.\(^6-11\) In a real 2DES with finite electron-layer thickness \( (w_e) \), the short-range component of the electron-electron interaction is weakened and this weakening results in a reduction of \( \Delta \).\(^7-11\) In fact, \( w_e \) is sufficiently large, a collapse of \( \Delta \) and a transition to an uncorrelated phase is expected.\(^10\) Experimentally, however, no measurements of the dependence of \( \Delta \) on \( w_e \) have been reported. Here we report such measurements for the first time.

We measured \( \Delta \) at \( \nu = \frac{1}{3} \) and \( \frac{2}{3} \) in a variable-width electron system realized in a selectively doped, parabolic, Al\(_x\)Ga\(_{1-x}\)As quantum well. The Al composition in the well is graded in such a way as to result in a quadratic dependence of the conduction-band edge on the distance from the well center. Owing to the self-consistent electrostatic potential, the electrons in the parabolic well screen the quadratic potential, and a system of nearly uniformly distributed electrons in a flat potential is obtained.\(^12,13\) In this system, \( w_e \) increases with increasing areal density \( (n_e) \) in the well so that the effective three-dimensional density and the Fermi energy \( (E_F) \) remain essentially constant.\(^12,13\) This is in contrast to the case of 2DES at heterointerfaces where \( w_e \) usually has a weak dependence on \( n_e \). We observe a dramatic decrease in the excitation energies for both \( \nu = \frac{1}{3} \) and \( \frac{2}{3} \) FQHE states as \( n_e \) increases, in sharp contrast to the commonly observed increase \( \Delta \) with \( n_e \) for 2DES at single heterojunctions.\(^4,5\) This dramatic decrease of \( \Delta \) implies a quenching of the FQHE in an electron system with sufficiently large \( w_e \) and small electric-subband separation.

The structure used in this study was grown on an undoped (100) GaAs substrate by molecular-beam epitaxy and mainly consists of a 3000-Å-wide, undoped, parabolically graded, Al\(_x\)Ga\(_{1-x}\)As well bounded on both sides by undoped (spacer) and doped layers of Al\(_x\)Ga\(_{1-x}\)As \( (y > x) \). The Al composition \( (x) \) is zero at the well center and is quadratically varied near the well center to produce a parabolic band profile.\(^12\) The spacer is 580 Å of undoped Al\(_{0.4}\)Ga\(_{0.6}\)As, and the doping consists of five \( \delta \) layers of Si (each with a density of \( 1.3 \times 10^{11} \) cm\(^{-2} \)) and separated by 30 Å. Here we report our results for two samples \( A \) and \( B \); the main difference between these is that the parabolic well of sample \( B \) has a larger curvature.

To measure the transport coefficients, contacts were made by alloying In in a H\(_2\) atmosphere at 400°C for 13 min. The sample was immersed in the tail of the mixing chamber of a top-loading dilution refrigerator, and a calibrated RuO\(_2\) resistance thermometer was used to measure temperature with an absolute accuracy of better than 5%. In our experiments we varied \( n_e \) in the well by a combination of careful cooling to cryogenic temperatures and illumination by a red-light-emitting diode. The magnetic field was provided by a superconducting solenoid. For electrical measurements we used the lock-in technique at a frequency of 6.8 Hz. Sample excitation current was in the range 1-10 nA, and no electron heating effects were observed for the reported results.

In Fig. 1 the transport data for sample \( A \) are shown for three different \( n_e \). The high-field data shown in the left panel of Fig. 1 exhibit the integral and fractional quantum Hall effects, with the positions (in \( B \)) of \( \rho_{xx} \) minima and \( \rho_{xy} \) plateaus (not shown here) for \( B \gtrsim 1 \) T being consistent with the \( n_e \) indicated in the figure. The
The longitudinal resistivity ($\rho_{xx}$) is shown for sample $A$ for the three different $n_s$ in the left and center panels. The measured mobility is $3.6 \times 10^5$ cm$^2$/Vs for all three $n_s$. The arrows in the left panels indicate the Landau-level filling factors at which the integral or fractional quantum Hall effect is observed. The right panel shows the Fourier power spectra of the oscillatory $\rho_{xx}$ vs $1/B$ data of the center panel. The arrows here indicate the expected positions of the frequencies (subband densities) according to our self-consistent calculations.

A high quality of the electron system in this structure is evidenced by the observed higher-order FQHE states at $\nu = \frac{1}{3}$, $\frac{2}{5}$, and $\frac{2}{7}$ in addition to the fundamental $\frac{1}{3}$ and $\frac{1}{5}$ states. With increasing $n_s$, however, the $\nu = \frac{1}{3}$ and $\frac{1}{5}$ states become weaker and the higher-order states disappear.

We first demonstrate quantitatively how $w_e$ in our samples increases with $n_s$. In Fig. 2 we show the results of our self-consistent calculations for the charge distribution and electric-subband energies in our structure. The calculations were performed by solving the Poisson and Schrödinger equations (for each $n_s$), and taking into account the exchange correlation via the local-density-functional approximation. As expected, the width $w_e$ (which we define as the full width at half maximum) of the electron layer increases with $n_s$ so that the effective 3D density as well as $E_F$ (at $B = 0$) remain essentially constant. Note also that with increasing $n_s$ and $w_e$, the separation between electric subbands decreases and more subbands become occupied.

The quantitative experimental evidence for the realization of the designed electron system is provided by an analysis of the observed Shubnikov-de Haas oscillations in $\rho_{xx}$ at low $B$ (center panels in Fig. 1). The frequencies of these oscillations, which can be determined from the Fourier transforms of the $\rho_{xx}$ vs $1/B$ data (right panels of Fig. 1), directly give the areas of the Fermi-surface cross sections (circles in our case), and therefore the areal electron densities for the different occupied electric subbands. To compare the results of the calcula-

(a) The calculated charge distribution for sample $A$ for different areal densities at $B = 0$. (b) The calculated electric-subband separations and the Fermi energy as a function of $n_s$ for the same sample. The dotted curve in (b) represents $E_2 - E_1$ if we assume only the lowest subband is occupied (see Ref. 14). (c) The calculated electron-layer width as a function of $n_s$ for both samples $A$ and $B$.
tion with the experiment, in the right panels of Fig. 1 we have indicated the calculated subband densities (frequencies) by vertical arrows. The very good agreement between the data and the results of the calculation verifies the realization of the electron systems indicated in Fig. 2.

We obtain $\Delta$ by fitting the temperature dependence of the $\rho_{xx}$ minima (Fig. 3) by $\rho_{xx} = \rho_0 \exp(-\Delta/2T)$. Similar to most of the previously reported data for the FQHE in 2DES at GaAs/AlGaAs heterojunctions, the data show an activated behavior only in a limited temperature range.\(^4\) We used the slopes of the dotted lines drawn through this intermediate temperature range (Fig. 3) to determine $\Delta$. The results are shown in Fig. 4. We also analyzed our data following the work of Boeinger et al.\(^4\) by fitting the data with an expression containing an activated term and a term arising from hopping conduction which dominates at low temperatures. The values of $\Delta$ obtained from the fittings are somewhat larger than those determined directly from the slopes of the straight lines. The qualitative behavior of $\Delta$ and especially its dependence on $n_s$, however, remain the same for both types of analyses.

To interpret the data, we first summarize what is known about $\Delta$ in FQHE. For an ideal 2DES, the theoretically expected behavior for $\Delta$ at $\nu = 1/3$ and $2/3$ is $\Delta \sim 0.1 e^2/\ell_0 \ll B^{1/2} \propto n_s^{1/2}$.\(^{6-11}\) Experimentally measured $\Delta$ for 2DES in GaAs/AlGaAs heterostructures, on the other hand, are substantially smaller than $0.1 e^2/\ell_0$.\(^4,5\) The discrepancy between the experiment and theory has been attributed to several factors which are not considered in the calculations for the ideal 2DES; these include the finite thickness of the electron layer,\(^7,8\) Landau-level mixing,\(^2\) and the existence of finite disorder in the nonideal 2DES.\(^15\)

Although the experimentally measured $\Delta$ in 2DES at the GaAs/AlGaAs interface are significantly smaller than $0.1 e^2/\ell_0$ and do not follow a $n_s^{1/2}$ (or $B^{1/2}$) dependence, experiments have shown that, for samples of roughly similar quality, $\Delta$ increases with $n_s$.\(^4,5\) This is in sharp contrast to our data summarized in Fig. 4(a). We believe that the drastic reduction of $\Delta$ with increasing $n_s$ observed in our structure is related to the increase of $w_e$ and, in part, to the decrease in the subband spacing.\(^16,17\) We firmly believe that disorder is not playing a significant role in the observed collapse of $\Delta$. The primary evidence for our conclusion is provided by the measured $\Delta$ themselves. If the observed decrease in $\Delta$ was caused by increasing disorder as $n_s$ (and $w_e$) increases, one would expect that (1) the $\nu = 1/3$ and $2/3$ gaps would start to decrease at the same $n_s$, and (2) for a given $n_s$, the $1/3$ gap would always be larger than the $2/3$ gap because the $2/3$ gap is measured at a magnetic field twice as large as the field at which the $1/3$ gap is measured.\(^4,5\) Figure 4(a) shows that our measured $\Delta$ behave quite differently. For a given sample, the gap for the $\nu = 1/3$ state starts to collapse at a lower $n_s$ than the $2/3$ gap does and, at high $n_s$, the $\nu = 1/3$ gap falls below the $2/3$ gap.

While the data of Fig. 4(a) indicate that disorder is not playing a major role in decreasing $\Delta$, Fig. 4(b) indicates that the layer thickness does. Figure 4(b) clearly shows that the $\nu = 1/3$ and $2/3$ data behave similarly when plotted against the relevant dimensionless parameters, i.e., $w_e$ measured in units of the magnetic length. The plots of Figs. 4(a) and 4(b) therefore provide convincing evidence that it is the layer thickness rather than disorder that is primarily responsible for changes in $\Delta$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3}
\caption{Temperature dependence of $\rho_{xx}$ at $\nu = 1/3$ for sample $B$ for three different $n_s$. The slopes of the dotted lines are used to determine the energy gap.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig4}
\caption{The dependence of the measured energy gap ($\Delta$) on electron density ($n_s$) and on the dimensionless parameter $w_e/\ell_0$ for $\nu = 1/3$ (closed symbols) and $2/3$ (open symbols). The closed and open circles are for sample $A$ and the rest of the data are for $B$. The estimated accuracy is about $\pm 10\%$ for $\Delta \approx 0.5$ K, and $\pm 30\%$ for $\Delta \lesssim 0.5$ K.}
\end{figure}
In our electron system $w_e$ [Fig. 2(c)] is comparable to or larger than the interelectron spacing ($\approx 400$ Å). The short-range component of the electron-electron interaction is therefore weakened, and we do indeed expect $\Delta$ to strongly depend on the layer thickness. Moreover, with increasing $w_e$, the subband energy spacings in our system decrease (Fig. 2) and become comparable with the measured $\Delta$ and certainly smaller than the Coulomb energy ($\sim e^2/\epsilon a_0$). Therefore, the effect of subband mixing on reducing $\Delta$ is also important. We note that in any physical electron system (such as ours) in a given material, the subband spacings and the layer thickness are fundamentally related; i.e., it is not possible to vary the layer thickness and at the same time keep the energy spacings fixed. An experimental study of the dependence of $\Delta$ on $w_e$ will therefore, in general, include the effects of both finite layer thickness and subband mixing.

The lack of any theoretical calculations for $\Delta$ in a system like ours, with a very wide electron-layer distribution and small subband separation, precludes us from a more quantitative comparison of our data with theory. It is worth mentioning, however, that the drastic reduction in $\Delta$ that we observe for a relatively small increase in $n_i$ or $w_e$, or a decrease in subband energy spacing, cannot be explained by a simple extrapolation of the existing calculations. Our observation, we believe, manifests the collapse of FQHE electron correlations expected in the theories of the FQHE as either the electron-layer thickness is increased or the electric-subband separation is decreased. We hope our results will stimulate future calculations of $\Delta$ for these structures so that a more quantitative comparison between experiment and theory can be made.

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Note added.—Motivated by our experimental results, He et al. have recently reported calculations for the FQHE in a clean (no disorder) system with large $w_e$. They find that the FQHE starts to collapse once $w_e/I_0 \sim 3$, and is practically destroyed when $w_e/I_0 \gtrsim 7$, in good agreement with our data [Fig. 4(b)].

14In a single-electron picture, at high magnetic fields ($\nu < 1$), the electrons occupy only the lowest subband. Assuming that only one subband is occupied, our self-consistent calculations show that the high-field subband separation decreases Idotd curve in Fig. 2(b), and also that the layer thickness decreases ($< 10\%$) with respect to the zero-field $w_e$. In a many-electron picture, however, the possibility of subband (and Landau-level) mixing exists.
15The effect of disorder has been considered only in a phenomenological manner [see, e.g., A. H. MacDonald, K. L. Liu, S. M. Girvin, and P. M. Platzman, Phys. Rev. B 33, 4014 (1985); A. Gold, Europhys. Lett. 1, 241 (1986); 1, 479(E) (1986); P. M. Platzman, Phys. Rev. B 39, 7985 (1989)].
16In our system, with increasing electron-layer thickness, there is the possibility of enhanced alloy scattering [K. Karrai, X. Ying, H. D. Drew, and M. Shayegan, Surf. Sci. 229, 515 (1990)]. However, the measured mobility ($\mu$) in our structure in the density range $5 \times 10^{10} < n_i < 10^{11}$ cm$^{-2}$ is nearly constant ($\approx 3.6 \times 10^7$ cm$^2$/V s). Therefore, we do not believe that disorder is responsible for the significant decrease in $\Delta$ that we observe in the same density range. For 2DES in GaAs/AlGaAs heterojunctions, a $\mu \sim n_i^2$ (where $0.5 \leq a \leq 1.5$) behavior is commonly observed. The enhancement of $\mu$ with $n_i$ is generally related to the increase in $E_F$. In our parabolic quantum well, however, $E_F$ is relatively independent of $n_i$ once the second subband starts to be populated (see Fig. 2). The fixed value of $\mu$ for $n_i \gtrsim 5 \times 10^{10}$ cm$^{-2}$ is therefore not surprising.
17The dramatic drop in $\Delta$ that we observe is at values of $n_i$, well above the $n_i$ where a minimum in $\Delta$ has recently been reported and attributed to spin-polarization effects [J. P. Eisenstein et al., Phys. Rev. B 41, 7910 (1990)]. Therefore, we do not believe that spin polarization is playing a dominant role here.