The Quantum Antidot Electrometer: Direct Observation of Fractional Charge

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A Quantum Antidot electrometer has been used in the first direct observation of the fractionally quantized electric charge. In this paper, after introductory remarks, we review recent experiments performed on the integer \( i = 1, 2 \) and fractional \( f = 1/3 \) quantum Hall plateaus extending over a filling factor range of at least 27%. On the integer plateaus we directly measure electron's charge, in Coulombs, with standard deviation of 0.7% and absolute accuracy of 0.035e. We further find the charge of the \( f = 1/3 \) Laughlin quasiparticles to be invariantly \( e/3 \), with standard deviation of 1.2%, independent of filling factor \( \nu \), tunneling current, and temperature.

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I. INTRODUCTION

The last two decades since the discovery of the integer \([1]\) and fractional \([2]\) quantum Hall effects have witnessed remarkable increase in our understanding of low-dimensional strongly correlated systems. Many surprising and many beautiful experiments were performed, new theoretical understanding was gained, new elegant and sophisticated techniques were developed, both experimental and theoretical. \([3]\) The 1985 and 1998 Nobel prizes in physics \([4]\) were awarded to von Klitzing, and Tsui, Stormer and Laughlin for the discoveries.

The most conspicuous aspect of the quantum Hall effect (QHE) is the constancy of the Hall conductance over a finite range of the filling factor \( \nu \). Indeed, this property defines the phenomenon of QHE, and the quantized value of the Hall conductance \( \sigma_{xy} \) of a particular QH state in units of \( e^2/h \) is a principal quantum number of that QH state \( i \) or \( f \), called “exact filling”. Specifically, the electric charge of the quasiparticles is expected to be determined by the relevant quantum numbers, including \( i \) or \( f \), and thus not expected to vary on a QH plateau as \( \nu \) is varied from the exact filling. \([5,6]\) The exactness of quantization of the Hall conductance is understood as a consequence of the gauge invariance of electromagnetic field and the exact quantization of the charge of electrons. \([5]\) A variation of Laughlin’s gedanken experiment considers a Corbino disc of two-dimensional (2D) electrons of areal density \( n \) with quantizing magnetic field \( B \) applied normal to the disc, so that Landau level filling is \( \nu = hn/eB \), see Fig. 1. \([7]\) At a low temperature \( T \) the system of electrons condenses into a QH state, integer or fractional, depending on \( \nu \) and on how much disorder is present. Let us first consider the case of the integer QHE, where the integer \( i = 1, 2, ... \) is the number of Landau levels occupied by electrons, and the exact filling occurs at \( \nu = i \). Now add adiabatically a flux quantum \( \phi_0 = h/e \) in the inner hole of the disc; while \( \phi_0 \) is added one electron per occupied Landau level is transferred between the inner and the outer edges of the disc (provided they are connected by a wire), but, since the flux is added in the hole, the state of the electron system must be exactly the same as that before flux was added (gauge invariance). Thus, in the steady state, we add \( \phi_0 \) every \( \delta t \), the magnetic flux through the Corbino ring center changes at the rate \( d\Phi/dt = \phi_0/\delta t \), and the Faraday-induced azimuthal voltage \( (\text{emf}) \) is \( V_\varphi = \int r d\varphi E_\varphi = -\phi_0/\delta t \) (we use \( E_\varphi = \partial A_\varphi/\partial t \)). On the other hand, the radial Hall current is \( I_r = -ie/\delta t \), the charge transferred in one Landau level per \( \delta t \) times the number of Landau levels occupied. Thus the Hall resistance \( R_{xy} = R_{r\varphi} = \phi_0/ie \) (we note that in 2D \( \rho_{xy} = R_{xy} \)). What is important here is that \( R_{xy} \) remains quantized exactly even as \( \nu \) is varied from the exact filling, because disorder localizes extra electrons or holes and thus the diagonal conductivity \( \sigma_{xx} = 0 \).

![FIG. 1. Gedanken experiment to demonstrate the exactness of quantization of Hall resistance \( R_{xy} = R_{r\varphi} \) on a QH plateau. Magnetic flux \( \Phi \) through the center of the Corbino 2D electron system ring changes at a constant rate thus inducing azimuthal electric field \( E_\varphi \). The loop integral of \( E_\varphi \) is \( V_\varphi \), which corresponds to the radial Hall current: \( V_\varphi = R_{r\varphi} I_r \).](image-url)
Our understanding of the fractional QHE is based on Laughlin’s many electron wave function, [8] which describes the \( f = \frac{1}{m+1} \) QH states. A particularly transparent generalization of the Laughlin’s theory has been developed by Jain, who showed that the fractional QHE at \( \nu \approx f = \frac{1}{2m+1} \) can be mapped onto the integer case using composite fermion theory. [9] A composite fermion is an electron bound to an even number \( 2p \) (\( p = 1, 2, \ldots \)) of vortices of the many-particle wave function. The binding results from Coulomb interaction between the electrons, and it has been shown that the exact FQH ground states are very close to those of the composite fermion theory. Also, the composite fermion theory predicts the hierarchy of the FQH states observed in nature. [10] Since the number of vortices of the many-particle wave function is equal to the number of the flux quanta \( \Phi_0 \), in mean field theory composite fermions can be thought of as electrons each binding \( 2 \) of the GaAs sample of thickness \( d \). This gate forms a part of the QAD, as electrons each binding \( 2\Phi_0 \) of applied \( B \). Thus composite fermions experience effective magnetic field \( B_{cf} = B - 2p\phi_0 \), and the filling of the pseudo “Landau levels” of composite fermions \( \nu_{cf} = \nu\phi_0 / \Phi_0 \) corresponds to \( \nu = n\phi_0 / B = \nu_{cf} / (2p\nu_{cf} \pm 1) \). For \( p = 1 \), for example, the FQHE of interacting electrons at \( f = \frac{1}{2m+1} \) looks like the IQHE of weakly interacting composite fermions at \( i = i_{cf} \), with \( i_{cf} \) pseudo “Landau levels” occupied by composite fermions.

II. TUNNELING VIA QUANTUM ANTIDOT

We reviewed the Laughlin’s gedanken experiment because the quantum antidot (QAD) experiments are a microscopic realization of interedge particle transfer, one at a time, when the flux through the antidot is varied. In the case of QAD, however, the external wire connecting the two edges of a QH sample is eliminated, the particle transport is accomplished by internal tunneling through the QH gap. The QAD electrometer is illustrated in Figs. 2 and 3. [11,12] The antidot is defined lithographically in a constriction between two front gates in a 2D electron system (2DES). The electrometer application is made possible by a large, global “back gate” on the other side of the GaAs sample of thickness \( d \). This gate forms a parallel plate capacitor with the 2DES. The antidot and the front gates create depletion potential hills in the 2DES plane and, in quantizing magnetic field \( B \), the QHE edge channels are formed following equipotentials where the density \( n \) is such that \( \nu = en/hB \) is equal to integer \( i \) or fractional \( f \) exact filling.

The edge channels on the periphery of the 2DES have a continuous energy spectrum, while the particle states of the edge channel circling the antidot are quantized by the Aharonov-Bohm condition. In the lowest Landau level the basis orbital

\[
\psi_m(r) = (2\pi 2^m m!)^{-1/2} r^m \exp(-\frac{r^2}{4\ell^2})
\]

with quantum number \( m \) encloses \( m\phi_0 \) magnetic flux, \( m = 0, 1, \ldots \). Here \( \ell = (\hbar/eB)^{1/2} \) is the magnetic length. In other words, the semiclassical area of the orbital \( \psi_m \) is \( S_m = m\phi_0 / B \). Analogous QAD-bound wave functions exist in each Landau level.

When the constriction is on a quantum Hall plateau, particles can tunnel resonantly via the QAD-bound states giving rise to quasiperiodic tunneling conductance \( G_T \) peaks, see Fig. 3. [13] A peak in \( G_T \) occurs when a QAD-bound state crosses the chemical potential and thus marks the change of QAD occupation by one particle. The experimental fact that the \( G_T \) peaks are observed implies that the charge induced in the QAD is quantized, that it comes in discrete particles occupying the antidot-bound states; the \( G_T \) peaks mark the change of the population of the QAD by one particle per peak. Measuring \( G_T \) as a function of \( B \) gives the area of the QAD-bound state through which the tunneling occurs, \( S = \phi_0 / \Delta B \), where \( \Delta B \) is the quasiperiod in magnetic field. On the \( \nu \approx i \) plateau there are \( i \) peaks per \( \Delta B \).
because \( i \) Landau levels are occupied. The above discussion is easy to generalize for \( f = \frac{i}{2p+1} \) FQH plateaus by considering “Landau levels” of composite fermions.

III. QUANTUM ANTIDOT ELECTROMETER

On a QH plateau the charge of quasiparticles localized in the interior of a 2D electron system is well defined. \([5,14]\) In the case of the integer QH plateau at \( \nu \approx i \) the quasielectrons are simply electrons in the Landau level \( i + 1 \), and the quasiholes are the holes in the \( i^{th} \) level. It is easy to understand some of the properties of FQHE quasiparticles using composite fermions. \([9,10]\) In the case of the FQH plateau at \( f = \frac{i}{2p+1} \) quasielectrons are composite fermions (an electron binding 2 electrons in a + 1 state); \( i \) is the Landau level. Ithasbeenpredicted that occupation of the FQH plateau at \( \nu = 2n \) is precisely equal to the 2D charge density induced by \( e / 3 \) quasielectrons. \([17]\) Thus, the charge of one particle \( q \) is directly given by the electric field needed to attract one more particle in the area \( S \): \( q = e_0 S \Delta V_{BG}/d \), where \( \Delta V_{BG} \) is the change of the global gate voltage between two consecutive conductance peaks. \([18]\) An absolute and more accurate determination of \( q \) uses direct measurement of area \( S = \phi_0/\Delta B \) for each QH plateau, as described in Sect. V.

We use low disorder GaAs heterojunction material where 2DES (density \( 1 \times 10^{11} \text{ cm}^{-2} \) and mobility \( 2 \times 10^6 \text{ cm}^2/\text{Vs} \)) is prepared by exposure to red light at 4.2 K. The antidot-in-a-constriction geometry (somewhat different from that of Refs. \([11,12]\)) was defined by electron
beam lithography on a pre-etched mesa with Ohmic contacts. After ≈150 nm chemical etching Au/Ti gate metallization was deposited in the etched trenches. Samples were mounted on sapphire substrates with In metal which serves as the global back gate. All data presented in this paper were taken at 12 mK bath temperature. Extensive cold filtering cuts the electromagnetic background incident on the sample up to 5×10⁻¹⁷ W, which allows us to achieve a record low effective electron temperature of 18 mK reported for a mesoscopic sample. [19,20]

Figure 5 shows the directly measured four-terminal $R_{xx}$ vs. $B$ data in the integer quantum Hall regime. We use $\nu_C$ to denote the filling factor in the constriction region between the front gates, and $\nu_B$ for the rest of the sample, “the bulk”, away from the constriction region. The front gates are biased negatively in order to bring the edges closer to the antidot to increase the amplitude of the tunneling peaks to a measurable level. This results in $\nu_C$ being smaller than $\nu_B$ in the bulk. A QHE sample with two $\nu_B$ regions separated by a lower $\nu_C$ region, if no tunneling occurs, has $R_{xx} = R_L \approx R_{xy}(\nu_C) - R_{xy}(\nu_B)$. The equality is exact, $R_L = \frac{h}{2e^2}(1/i_C - 1/i_B)$, if both filling factors $\nu$ and $\nu_B$ are on a plateau, $\nu_C \approx i_C$ and $\nu_B \approx i_B$, which can be achieved by adjusting the front gate voltages. In such situation the Hall resistances of all regions acquire quantized values. Thus, several $R_L$ plateaus (neglecting tunneling peaks) are seen in Fig. 5.

In some data (cf. Fig. 5) we observe both $R_{xx}$ peaks for $\nu_C < i_C$ (“back scattering”) and dips for $\nu_C > i_C$ (“forward scattering”). [13,21] Details of this behavior will be presented elsewhere. [22] The tunneling peaks are superimposed on the smooth $R_L$ background, and we calculate tunneling conductance $G_T$ as described previously. In essence, when $R_L$ is quantized, the edge network model calculation gives [11,19]

$$G_T = \frac{R_{xx} - R_L}{R_H - R_L(R_{xx} - R_L)}$$

where constriction Hall resistance $R_H = h/i_C e^2$. In the weak tunneling limit $G_T << \frac{e^2}{\hbar}$ this expression reduces to $G_T = (R_{xx} - R_L)/R_H^2$, thus $G_T$ line shape is directly proportional to the directly measured resistance peaks $R_{xx} - R_L$ for weak tunneling.

For the QAD sample presented in this work, from the spacing between the tunneling peaks $\Delta B$ we immediately obtain the antidot radius: $r = \sqrt{\hbar \nu / \pi \Delta B} \approx 345 \text{ nm}$, the quantum number of the state at the chemical potential $m = \Delta B / B \approx 60, 160, \text{ and } 770$ for the centers of the $\nu_C = 2, 1, $ and $1/3$ plateaus, respectively (see Figs. 6 - 8). From the temperature dependence of $G_T$ peak width [22,23] we obtain energy spacing $\Delta E \approx 100 \mu\text{eV}$.

V. INVARIANCE OF QUASIPARTICLE CHARGE

FQH quasiparticles are the elementary charged excitations of the FQH condensate, a distinct state of matter that is necessarily separated from all noninteracting electron states by a phase transition. Therefore, FQH quasiparticles are composite objects vastly different from electrons, and are not adiabatic images of electrons as are quasiparticle excitations of metals and band insulators (such as IQHE states).

Although six years have passed since the first direct observation of $\frac{1}{3} e$ and $\frac{1}{5} e$ particles in quantum antidot electrometer experiments, [11,12] one crucial aspect of theory remained untested: the invariance of charge at $\nu$ far from exact filling. In this section we review recent experiments performed on the integer $i = 1, 2$ and fractional $f = \frac{1}{3}$ quantum Hall plateaus which extend over a filling factor $\nu$ range of 27% to 45%. [24] The charge of the QAD-bound quasiparticles has been measured to be constant, independent of $\nu$ over the entire plateau extent, with relative accuracy of ±1.2% and absolute accuracy of 3.5%. In addition, we observe no variation of the quasiparticle charge upon variation of temperature or applied current, in the experimentally accessible range.

Figures 6 - 8 show the directly measured four-terminal $R_{xx}$ vs. $B$ data for three QAD plateaus: $i = 2, i = 1$ and $f = \frac{1}{3}$. In this Section $\nu$ denotes the filling factor in the constriction region. Thus, several $R_L$ plateaus (neglecting tunneling peaks) are seen in Figs. 6 - 8.
The tunneling peaks are superimposed on the smooth $R_L$ background, and we calculate $G_T$ as described in Section IV. [11,19] In some data (Figs. 6 and 8) we observe both $R_{xx}$ peaks for $\nu < i$ ("back scattering") and dips for $\nu > i$ ("forward scattering"). [13,21] At several $\nu$ on each plateau we took high resolution $B$-sweeps at $V_{BG} = 0$.

FIG. 6. $R_{xx}$ vs. magnetic field $B$ for the antidot filling $\nu$ on the $i = 2$ plateau. The upper panels (a) - (c) give the tunneling conductance measured both as a function of $B$ (back gate voltage is held constant $V_{BG} = 0$), and as a function of $V_{BG}$ ($B$ is held constant, shown by arrows in the lower panel). The $G_T$ vs. $V_{BG}$ curves in panels (a) - (c) are offset vertically.

FIG. 7. $R_{xx}$ vs. magnetic field $B$ for the antidot filling $\nu$ on the $i = 1$ plateau. The upper panels (a) - (c) give the tunneling conductance measured both as a function of $B$ (back gate voltage $V_{BG} = 0$), and as a function of $V_{BG}$ ($B$ is held constant).
and, having put the superconducting magnet in the persistent current mode to fix $B$, we took corresponding sweeps of $V_{BG}$. Representative $G_T$ vs. $B$ and $V_{BG}$ data are shown in the upper panels of Figs. 6 - 8. [25]

Note that the negative $V_{BG}$ axis direction corresponds to the increasing $B$. This is so because increasing $B$ results in relative depopulation of QAD independent of the sign of the charge of particles: $\psi_m$ move closer to the center of the QAD, that is, more states move above the chemical potential and become unoccupied. Negative $V_{BG}$ also depopulates the QAD if QAD-bound particles have negative charge. Thus the QAD electrometer measures not only the magnitude of $q$, but also its sign.

The magnitude of the charge $q$ of the QAD-bound particles is then determined from

$$q = \frac{\epsilon \epsilon_0 \phi_0}{d} \cdot \frac{\Delta V_{BG}}{\Delta B}$$

in Coulombs, (3)

using the low-temperature GaAs dielectric constant $\epsilon = 12.74$ [26] and the measured thickness of the sample $d \approx 0.430$ mm. The average of the eleven values obtained for the $i = 1, 2$ plateaus $\langle q \rangle_{\text{integer}} = 0.9651e$ is off by 3.5%, more than the standard deviation of 0.0070e for the combined data (similar to the results from another electrometer device reported in Refs. [11,12]). We then normalize values of $q$ by setting $\langle q \rangle_{\text{integer}} = e$. Thus determined $q$ are given for (a) - (c) data in Figs. 6 - 8, and summarized in Fig. 9. The striking feature of the data of Fig. 9 is that the values of $q$ are constant to a relative accuracy of at least $\pm 1.2\%$ throughout the plateau regions where it was possible to measure the particle charge. The range of $\nu$ is about 45% for the $i = 2$ plateau, 27% for the $i = 1$ plateau (the combined normalized $\nu/i$ range is 57%), and also 27% for the $f = \frac{1}{3}$ plateau.

FIG. 8. $R_{xx}$ vs. magnetic field $B$ for the antidot filling $\nu$ on the $f = \frac{1}{3}$ plateau. The upper panels (a) - (c) give the tunneling conductance measured both as a function of $B$ (back gate voltage $V_{BG} = 0$), and as a function of $V_{BG}$ ($B$ is held constant).

FIG. 9. Summary of the measured QAD-bound particle charge $q$ in units of $e$. The horizontal dashed lines give the values $e$ and $e/3$. The solid lines have the unit slope. It is quite evident that the ratio $\Delta V_{BG}/\Delta B$ (right axis) is not proportional to $\nu$. Therefore, $\nu$ is about 45% for the $i = 2$ plateau, 27% for the $i = 1$ plateau (the combined normalized $\nu/i$ range is 57%), and also 27% for the $f = \frac{1}{3}$ plateau.
We also note that the tunneling current $I_t \approx IG_T/\sigma_{xy}$ is proportional to $G_T$ and thus varies much for peaks of different amplitude, see Fig. 8. Here $I$ is the applied current used to measure $R_{xx}$; for example, the $f = \frac{1}{3}$ plateau data of Fig. 8 was taken with $I = 50$ pA, and we have measured $\Delta V_{BG}/\Delta B$ with $I$ up to 1 nA, which, combined with the variation in $G_T$, gives the range of $5 \times 10^{-13}$ A $\leq I_t \leq 2 \times 10^{-10}$ A. Furthermore, $\Delta V_{BG}/\Delta B$ has been measured in the temperature range 12 mK $\leq T \leq 70$ mK. [19,20] Under these conditions we observe no change in the value of $q = e/3$ within our experimental accuracy of a few percent. [27] Thus we conclude, based on our experimental results, that the charge of the QH quasiparticles is indeed a well defined quantum number characterizing a particular QHE state.

VI. ACKNOWLEDGMENTS

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VII. REFERENCES

[14] The excitation spectrum of QH edge channels is gapless, thus excitations of arbitrary charge can be created; e.g., X.-G. Wen, Intl. J. Mod. Phys. B 6, 1711 (1992). These edge excitations are a sort of 1D charge density fluctuations in the direction perpendicular to the edge, and generally are not Laughlin quasiparticles. A weakly coupled QAD serves as a “charge filter” that selects bulk Laughlin quasiparticles for interedge resonant tunneling.
[16] Even on a QHE plateau it is possible to change 2DES density in the “incompressible” interior with a gate, so long as it is done slowly, on the time scale of $\tau = RC \approx \epsilon_0 L^2/(d\sigma_{xx})$, where $L$ is the size of 2DES, because of finite diagonal conductivity $\sigma_{xx}$ at a finite temperature.
[18] A small front gate that defines the antidot (as in J. Franklin et al., Surf. Sci. 361/362, 17 (1996)) clearly does not produce a uniform $E_z$.
[22] I. Karakurt et al., to be published.
[25] Unintentional impurities near the antidot can change their charge state as $B$ or $V_{BG}$ are swept. Such events, often reproducible, cause “phase slips” of the $G_T$ peaks; we exclude such data from the charge analysis.
[27] These our results are in stark disagreement with recent reports of shot noise power experiments in constrictions without antidots, which, interpreted as a measurement of fractional charge, show variations of $q$ by factors of up to three and even six depending on all: $\nu$, $I_t$, and $T$. See, e.g., D. Glattli et al., Physica E 6, 22 (2000); T. Griffiths et al., cond-mat/0007264.