Flux Period Scaling in the Laughlin Quasiparticle Interferometer

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Observation of Aharonov-Bohm superperiod

- Under suitable magnetic field, the island filling factor is 2/5, surrounded by 1/3 QH fluid
- The observed Aharonov-Bohm (A-B) oscillation flux period $5h/e$ implies fractional statistics of Laughlin quasiparticles
Determine the flux period of A-B superperiod

- Directly measured is the magnetic field period $\Delta_B$. To determine the flux period, the area $S$ of the A-B orbit must be known

\[ \Delta \Phi = S \Delta_B \]

- To evaluate $\Delta \Phi$:
  - Estimate the tunneling rate between the outer and inner rings†
  - Self-consistent classical electrostatics of the island electron density profile †
  - Flux period scaling for integer and fractional filling factors

† Camino, Zhou, Goldman, PRB 72, 075342 (2005), PRL 95, 246802 (2005)
Island electron density profile

- Edge depletion model (Gelfand & Halperin) and the front gate depletion model (Chklovskii et al.)

- The fitting parameter is the depletion length $W$ ($V_{FG}=0$), fixed by requiring the curve goes through the closed blue circle.

- Matching the density of the 2/5 QH state to the curve $\Rightarrow$ the inner ring radius (A-B orbit radius, red circle)

$$\frac{n(\frac{2}{5})}{n(\frac{1}{3})} = \frac{2}{5} / \frac{1}{3} = 1.2$$

- Front gate voltage modifies:
  - overall island electron density
  - confining potential near the edge $\Rightarrow$ position of edge channel

- These two effects are related to each other.
Sample parameters

<table>
<thead>
<tr>
<th>Sample</th>
<th>M97Bm</th>
<th>M61Dd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Litho. radius (nm)</td>
<td>1050</td>
<td>1300</td>
</tr>
<tr>
<td>Etch depth (nm)</td>
<td>140</td>
<td>82</td>
</tr>
<tr>
<td>Electron density (cm&lt;sup&gt;-2&lt;/sup&gt;)</td>
<td>1.2×10&lt;sup&gt;11&lt;/sup&gt;</td>
<td>9.7×10&lt;sup&gt;10&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

- Sample was placed in a dilution refrigerator with base temperature ~ 10 mK
- Current: ac 5.4 Hz, 50 ~ 200 pA
- Voltage detected by a lock-in-phase technique
- Cold filtering reduces electromagnetic noise on the sample to ≤ 5×10<sup>-17</sup> W
- A-B oscillations measured both in the integer case (f=1 for M97Bm and M61Dd) and the fractional case (f=2/5 embedded in f=1/3 for M97Bm)
Front gate voltage dependence of A-B oscillations

- Increasing negative voltage $V_{FG}$ decreases the island electron density, shifting the A-B oscillations to lower magnetic field.

- B-field period $\Delta_B$ linear with $V_{FG}$ for moderate bias $|V_{FG}| \leq 300$ mV.

$V_{FG}$ modifies the overall island electron density

$V_{FG}$ modifies the position of edge channel.
2D electron disk capacitance model to determine $\Delta \Phi$

- Treat the 2DES inside the A-B orbit (~2000 electrons) as a conducting disk of radius $r$, its capacitance $C \propto r$, ignoring a slowly varying $\log(r)$ term
- Area of A-B orbit $S$, $\Delta_B$ the magnetic field period, $\Delta \Phi$ the flux period
- Express front gate voltage $V_{FG}(1e)$ to attract charge $1e$ into the A-B area via directly measured quantities:

  Differentiate $\Delta \Phi = S \Delta_B$ with respect to $V_{FG}$ under fixed filling factor $f$

  \[
  \left( \frac{d\Delta_B}{dV_{FG}} S + \frac{dS}{dV_{FG}} \Delta_B \right) \bigg|_f = 0 \quad \Rightarrow \quad \frac{dS}{dV_{FG}} = - \frac{d\Delta_B}{dV_{FG}} \frac{S}{\Delta_B}
  \]

  Define $N_\Phi$ - the number of flux quanta per period: $\Delta \Phi = N_\Phi h/e$, then $S = N_\Phi h/e \Delta_B$, and

  \[
  \frac{dS}{dV_{FG}} = - \frac{d\Delta_B}{dV_{FG}} \frac{N_\Phi h}{e \Delta_B^2}
  \]  

  (1)
2D electron disk capacitance model

The front gate voltage change to attract charge 1e into the A-B area $V_{FG}(1e)$

OTOH, 1e occupies area $S(1e)=1/n$

In QH regime $S(1e)=\hbar/eB_1$, where $B_1=\hbar n/e$, the field when $\nu=1$

Linearize LHS of Eq. (1)

$$\frac{dS}{dV_{FG}} = \frac{S(1e)}{V_{FG}(1e)}$$

Substitute $S(1e)=\hbar/eB_1$

$$\frac{\hbar}{eB_1 V_{FG}(1e)} = -\frac{d\Delta_B}{dV_{FG}} \frac{N_\Phi \hbar}{e \Delta_B^2}$$

Solve for $V_{FG}(1e)$

$$V_{FG}(1e) = -\frac{\Delta_B^2}{(d\Delta_B/dV_{FG}) N_\Phi B_1}$$

Thus, $V_{FG}(1e)$ is expressed through directly measured quantities

Use it to determine $N_\Phi$
Results of analysis

- C~r \implies \text{expect } rV_{FG}(1e) = \text{const, independent of } f \text{ and A-B area}

<table>
<thead>
<tr>
<th>Sample</th>
<th>M97Bm (f=1)</th>
<th>M97Bm (2/5 in 1/3)</th>
<th>M61Dd (f=1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>\Delta_B(V_{FG}=0), mT</td>
<td>2.907</td>
<td>21.40</td>
<td>1.872</td>
</tr>
<tr>
<td>d\Delta_B/dV_{FG}, mT/V</td>
<td>-1.37</td>
<td>-12.6</td>
<td>-1.22</td>
</tr>
<tr>
<td>B_1, T</td>
<td>3.92</td>
<td>3.92</td>
<td>2.53</td>
</tr>
<tr>
<td>V_{FG}(1e), mV</td>
<td>1.58</td>
<td>1.86</td>
<td>1.14</td>
</tr>
<tr>
<td>r, nm</td>
<td>673</td>
<td>555</td>
<td>839</td>
</tr>
<tr>
<td>rV_{FG}(1e), V\cdot nm</td>
<td>1.06</td>
<td>1.03</td>
<td>0.956</td>
</tr>
</tbody>
</table>

- Assuming N_\Phi=5 for the case 2/5 embedded in 1/3 QH fluid, products rV_{FG}(1e) are equal within ±5%
- Assuming N_\Phi=5/2 gives rV_{FG}(1e)=0.728, not consistent with the f=1 values

**Flux period 5h/e fits the results best!**

- The flux scaling analysis does not require the island electron density model, but these two methods give consistent results
Conclusion

• From the experimental data, we find the value of $rV_{FG}(1e)$ is constant within 10% for different island filling factors.
• The results rule out the possible values for flux periods of $5h/2e$ and less.
• Flux period $5h/e$ fits the experimental data best.
• Flux period scaling for integer and fractional filling factors provides an independent method to determine the flux period from directly measured experimental quantities.