Dependence of the Fractional Quantum Hall Edge
Critical Exponent on the Range of Interaction

Recent experiments on external electron tunneling into an edge of a fractional quantum Hall (FQH) system find a striking current-voltage power law behavior \( I \propto V^\alpha \), with exponent \( \alpha = 2.7 \) on the \( f = \frac{1}{2} \) FQH plateau [1,2]. Such power law \( I-V \) characteristic with \( \alpha = 3 \) was predicted for electron tunneling into an edge channel at the boundary of the \( f = \frac{1}{2} \) FQH system: the low-energy dynamics is effectively 1D, and field-theoretic descriptions of edge channels as chiral Luttinger liquids have been developed [3]. The equality \( \alpha = 2i + 1 \) for a FQH state at \( f = \frac{1}{2} \) \((i = 1, 2, 3, \ldots)\) has been demonstrated theoretically in the disk geometry for the Laughlin wave function \( \Psi_L \), which is known to be the exact ground state for certain short-range interactions. The experiments consistently obtain values of \( \alpha < 2i + 1 \), however [2].

Here we report results of a large numerical study of the microscopic structure of the FQH edge at \( f = \frac{1}{2} \). To this end, we diagonalize the interaction Hamiltonian in the disk geometry for up to \( N = 12 \) spin-polarized electrons restricted to lowest Landau level. We construct numerically [4] the Laughlin \( \Psi_L \) and “Coulomb” \( \Psi_C \) wave functions as the ground states of the short-range and Coulomb Hamiltonians, respectively, for the total angular momentum \( M = \frac{2}{3}N(N - 1) \), which gives filling \( f = \frac{1}{2} \) in the thermodynamic limit \( N \to \infty \). In particular, we have obtained occupation numbers \( \rho(m) \) of the angular momentum basis orbitals

\[
\psi_n(r, \theta) = (2\pi 2^m m!)^{-1/2} r^m \exp(i m \theta - r^2/4),
\]

where radius \( r \) is in units of magnetic length \( \ell = \sqrt{n}/eB \). The Hilbert space is restricted by consideration of orbitals with angular momentum \( m \leq m_{\text{max}} \) only. For the Hal-dane \( V_1 \) short-range interaction, the Laughlin \( \rho_L(m) \) with \( m > m_{\text{max}} \) is equal to \( 3(N-1) \) vanish identically; for Coulomb interaction good convergence of \( \rho_C(m) \) is obtained by \( m_{\text{max}} = m_{\text{max}}^L + 5 \) [5]. For example, for \( N = 12 \), the largest \( f = \frac{1}{2} \) FQH system studied, \( M = 198 \) and the size of the Hilbert space is 15 293 119 for \( m_{\text{max}} = 35 \). Details of this study will be published elsewhere [6].

As has been demonstrated by Wen [3], the critical exponent \( \alpha \) is equal to the ratio of the occupation numbers

\[
\alpha = \rho(m_{\text{max}}^L - 1)/\rho(m_{\text{max}}^L)
\]

for the Laughlin state \( \Psi_L \) on the disk. Wen has also argued that this relationship must hold for any interaction, so long as the FQH state at the same filling \( f \) exists, and is not unique for the Laughlin wave function. In Fig. 1 we present the ratio of the occupation numbers for both \( \Psi_L \) (short-range interaction) and \( \Psi_C \) (true Coulomb interaction) for \( N = 3 \) to 12. As expected, we obtain \( \alpha_L = 3 \) to machine accuracy for \( \Psi_L \). For Coulomb-interacting electrons, the ratio is always less than 3, and an extrapolation to the thermodynamic limit gives \( \alpha_C = 2.62 \). While we do not know whether the extrapolation shown in Fig. 1 holds for \( N > 12 \), certain other systematic behavior present in the numerical data [6] allows us to project that in the \( N \to \infty \) limit \( 2.58 \leq \alpha_C \leq 2.75 \), and is definitely less than 3.

Thus we propose that the deviation of the experimental \( \alpha \) from the predicted \( \alpha_L \) values is not an artifact and is not due to corrections such as finite bulk diagonal conductivity, disorder, or a variation of the electron density in the sample. Rather, we propose, the effect has a fundamental origin: the value of the critical exponent is not universal but depends on the range of particular interaction. This work was supported in part by the NSF.

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[5] The sum of \( \rho_C(m) \) with \( m > m_{\text{max}} \) is less than 0.01/\( N \). This means that fixing total angular momentum \( M = \frac{2}{3}N(N - 1) \) selects the \( f = \frac{1}{2} \) state for Coulomb interaction, too: it fixes average density on the disk \( \langle \rho_C \rangle \approx \rho_L \) for \( r < r_{\text{edge}} \) to better than 10^-3 for any \( N \).