FRACTIONAL QUANTUM HALL EFFECT

A game of five halves

Proof that the delicate 5/2 fractional quantum Hall state survives constriction within a quantum point contact paves the way to realizing an experimental platform for exploring the bizarre world of non-abelian particle statistics.

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In two dimensions, the laws of physics permit the existence of particles known as anyons, whose fractional statistics are intermediate between those governing bosons and fermions. When two particles of a system of bosons are exchanged, the phase of the system remains unchanged, whereas for a system of fermions it changes by exactly π. Swapping two anyons, however, results in a phase factor intermediate between zero and π, the precise value being characteristic of the species of the particles. Just as Bose–Einstein condensation and the Pauli exclusion principle are two important consequences of the exchange of bosons and fermions, respectively, the more exotic behaviour of anyons also has profound implications. The exchange of all bosons and fermions is commutative — so that the outcome of multiple particle exchanges is independent of the order in which they occur — the statistics of which is referred to as abelian, after the nineteenth-century Norwegian mathematician Niels Henrik Abel. Indeed, this is also true for most anyons, but not for all. For come species of anyons, exchange is non-commutative, order dependent and is described by non-abelian statistics. Topological quantum computation performed by braiding non-abelian anyons has been proposed as a way of implementing intrinsically fault-tolerant quantum computation. And non-abelian gauge theories have become a fundamental part of the Standard Model of particle physics. Yet to date, no system has been confirmed to include such non-abelian anyonic particles. On page 561 of this issue, Jeffrey B. Miller and colleagues[1] take an important step towards demonstration of just such a system, through the fractional quantum Hall (FQH) effect.

The elementary quasiparticles that arise in the FQH electron fluids are known as Laughlin quasiparticles. In most cases, they exhibit abelian anyonic behaviour, including those in states associated with odd-denominator FQH filling fractions. But when a FQH plateau with filling fraction 5/2 was unexpectedly observed[2] in a two-dimensional electron gas sample of unprecedented quality, it became clear that the even-denominator state does not fit into the standard description. Yet despite the great strides that have been made in the fabrication of such samples in the two decades since this observation was made, the delicate 5/2 state remains elusive, and only arises in a few of the most pure GaAs-based materials, and even then only in relatively large samples.

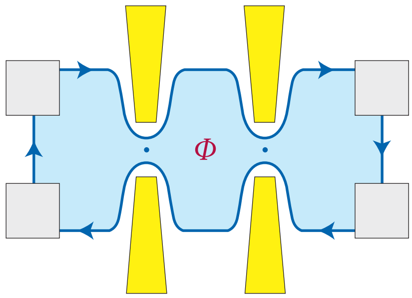

Figure 1 An Aharonov–Bohm quasiparticle interferometer for exploring non-abelian statistics. The two wide constrictions each defined by two nearby metal gates (gold fingers) separate a 2D electron island from a bulk 2D electron gas contacted by a series of four electrodes (silver squares). A magnetic field quantizes the electron gas, which develops counterpropagating quasiparticle edge channels. As quasiparticles circulate around these channels, they undergo quantum interference, which affects the measured conductance of the constrictions. The magnetic flux (Φ) passing through the electron island modulates the phase of the quasiparticles as they pass around the ring, and thereby how they interfere. As a result, varying this flux gives rise to periodic oscillations in the conductance — known as the Aharonov–Bohm effect. For fermions (such as electrons) or bosons (such as Cooper pairs in superconductors), the period of the oscillations is determined only by the value of their charge. For abelian anyons the period is determined by their charge and by how many are being encircled (that is, the number present on the island). But for non-abelian quasiparticles, more complex and exotic behaviour is expected.

The apparent fragility of the 5/2 state and the limited experimental data on it has made it difficult to develop a consistent theoretical framework with which to describe it. Several attempts have been made, the most intriguing being that based on the Moore–Read non-abelian many-electron wave function[3,4], though even this is not universally accepted as being valid. The 5/2 FQH gap separating the ground state from the charged excitations is small (up to 500 mK so far) and several competing gapped quantum Hall and other non-gapped ground states (such as composite fermion metal or charge-density wave states) are known to be very close in energy. In few-electron numerical calculations, the precise mathematical form used to describe the electron–electron interaction determines which of these represents the ground state of the system. Yet even this is difficult to establish owing to a number of complicating factors that arise in the real world, including the modifying influence of the finite thickness of the 2D electron layer; screening by nearby elements of the GaAs samples such as doping layers and metal gates; and not least by the effects of residual material disorder. Moreover, this sensitivity to competing ground states can cause the system to segregate into multiple phases, with different puddle-like regions dominated by different quasiparticle species with dissimilar behaviour. Such separations are likely to be made even worse by inhomogeneities introduced by external disorder, and the confinement potentials needed to define the micrometre-scale devices built to study 5/2 FQH states.

Consequently, for the large samples in which the 5/2 FQH fluid has been observed, it is not possible to deduce its ground state with any certainty, nor to predict whether a phase transition to a different ground state would occur under the effect of the confining potential of the constrictions in the 2D electron plane. Through meticulous attention to the material quality of their samples,