The Quantum Antidot Electrometer: Direct Observation of Fractional Charge

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A Quantum Antidot electrometer has been used in the first direct observation of the fractionally quantized electric charge. In this paper, after introductory remarks, we review recent experiments performed on the integer $i = 1, 2$ and fractional $f = 1/3$ quantum Hall plateaus extending over a filling factor range of at least 27%. On the integer plateaus we directly measure electron's charge, in Coulombs, with standard deviation of 0.7% and absolute accuracy of $0.035e$ We further find the charge of the $f = 1/3$ Laughlin quasiparticles to be invariantly $e/3$, with standard deviation of 1.2%, independent of filling factor ν, tunneling current, and temperature.

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I. INTRODUCTION

The last two decades since the discovery of the integer [1] and fractional [2] quantum Hall effects have witnessed remarkable increase in our understanding of low-dimensional strongly correlated systems. Many surprising and many beautiful experiments were performed, new theoretical understanding was gained, new elegant and sophisticated techniques were developed, both experimental and theoretical. [3] The 1985 and 1998 Nobel prizes in physics [4] were awarded to von Klitzing, and Tsui, Stormer and Laughlin for the discoveries.

The most conspicuous aspect of the quantum Hall effect (QHE) is the constancy of the Hall conductance over a finite range of the filling factor ν. Indeed, this property defines the phenomenon of QHE, and the quantized value of the Hall conductance σ_{xy} of a particular QH state in units of e^2/h is a principal quantum number of that QH state i or f, called "exact filling". Specifically, the electric charge of the quasiparticles is expected to be determined by the relevant quantum numbers, including i or f, and thus not expected to vary on a QH plateau as ν is varied from the exact filling. [5,6] The exactness of quantization of the Hall conductance is understood as a consequence of the gauge invariance of electromagnetic field and the exact quantization of the charge of electrons. [5] A variation of Laughlin's gedanken experiment considers a Corbino disc of two-dimensional (2D) electrons of areal density n with quantizing magnetic field B applied normal to the disc, so that Landau level filling is $\nu = hn/eB$, see Fig. 1. [7] At a low temperature T the system of electrons condenses into a QH state, integer or fractional, depending on ν and on how much disorder is present. Let us first consider the case of the integer QHE,

where the integer $i = 1, 2, ...$ is the number of Landau levels occupied by electrons, and the exact filling occurs at $\nu = i$. Now add adiabatically a flux quantum $\phi_0 = h/e$ in the inner hole of the disc; while ϕ_0 is added one electron per occupied Landau level is transferred between the inner and the outer edges of the disc (provided they are connected by a wire), but, since the flux is added in the hole, the state of the electron system must be exactly the same as that before flux was added (gauge invariance). Thus, in the steady state, we add ϕ_0 every δt, the magnetic flux through the Corbino ring center changes at the rate $d\Phi/dt = \phi_0/\delta t$, and the Faraday-induced azimuthal voltage (*emf*) is $V_\varphi = \oint r d\varphi E_\varphi = -\phi_0/\delta t$ (we use $E_\varphi = -\partial A_\varphi/\partial t$). On the other hand, the radial Hall current is $I_r = -ie/\delta t$, the charge transferred in one Landau level per δt times the number of Landau levels occupied. Thus the Hall resistance $R_{xy} = R_{r\varphi} = \phi_0/ie$ (we note that in 2D $\rho_{xy} = R_{xy}$). What is important here is that R_{xy} remains quantized exactly even as ν is varied from the exact filling, because disorder localizes extra electrons or holes and thus the diagonal conductivity $\sigma_{xx} = 0$.
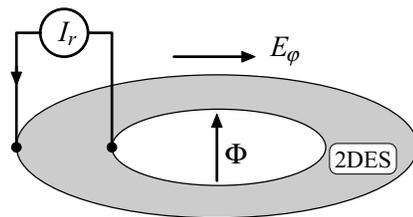

FIG. 1. Gedanken experiment to demonstrate the exactness of quantization of Hall resistance $R_{xy} = R_{r\varphi}$ on a QH plateau. Magnetic flux Φ through the center of the Corbino 2D electron system ring changes at a constant rate thus inducing azimuthal electric field E_φ. The loop integral of E_φ is V_φ, which corresponds to the radial Hall current: $V_\varphi = R_{r\varphi}I_r$.

Our understanding of the fractional QHE is based on Laughlin's many electron wave function, [8] which describes the $f = \frac{1}{2i \pm 1}$ QH states. A particularly transparent generalization of the Laughlin's theory has been developed by Jain, who showed that the fractional QHE at $\nu \approx f = \frac{i}{2pi \pm 1}$ can be mapped onto the integer case using composite fermion theory. [9] A composite fermion is an electron bound to an even number $2p$ $(p = 1, 2, ...)$ of vortices of the many-particle wave function. The binding results from Coulomb interaction between the electrons, and it has been shown that the exact FQH ground states are very close to those of the composite fermion theory. Also, the composite fermion theory predicts the hierarchy of the FQH states observed in nature. [10] Since on the average, in an area, the number of vortices of the many-particle wave function is equal to the number of the flux quanta ϕ_0, in mean field theory composite fermions can often be thought of as electrons each binding $2p\phi_0$ of applied B. Thus composite fermions experience effective magnetic field $B_{cf} = B - 2pn\phi_0$, and the filling of the pseudo "Landau levels" of composite fermions $\nu_{cf} = n\phi_0/|B_{cf}|$ corresponds to $\nu = n\phi_0/B = \nu_{cf}/(2p\nu_{cf} \pm 1)$. For $p = 1$, for example, the FQHE of interacting electrons at $f = \frac{i}{2i+1}$ looks like the IQHE of weakly interacting composite fermions at $i = i_{cf}$, with i_{cf} pseudo "Landau levels" occupied by composite fermions.

II. TUNNELING VIA QUANTUM ANTIDOT

We reviewed the Laughlin's gedanken experiment because the quantum antidot (QAD) experiments are a microscopic realization of interedge particle transfer, one at a time, when the flux through the antidot is varied. In the case of QAD, however, the external wire connecting the two edges of a QH sample is eliminated, the particle transport is accomplished by internal tunneling through the QH gap. The QAD electrometer is illustrated in Figs. 2 and 3. [11,12] The antidot is defined lithographically in a constriction between two front gates in a 2D electron system (2DES). The electrometer application is made possible by a large, global "back gate" on the other side of the GaAs sample of thickness d. This gate forms a parallel plate capacitor with the 2DES. The antidot and the front gates create depletion potential hills in the 2DES plane and, in quantizing magnetic field B, the QHE edge channels are formed following equipotentials where the density n is such that $\nu = en/hB$ is equal to integer i or fractional f exact filling.

The edge channels on the periphery of the 2DES have a continuous energy spectrum, while the particle states of the edge channel circling the antidot are quantized by the Aharonov-Bohm condition. In the lowest Landau level the basis orbital

$$\psi_m(r) = (2\pi \, 2^m \, m!)^{-1/2} \, r^m \exp(-\frac{r^2}{4 \, \ell^2}) \qquad (1)$$

with quantum number m encloses $m\phi_0$ magnetic flux, $m = 0, 1, ...$. Here $\ell = (\hbar/eB)^{1/2}$ is the magnetic length. In other words, the semiclassical area of the orbital ψ_m is $S_m = m\phi_0/B$. Analogous QAD-bound wave functions exist in each Landau level.
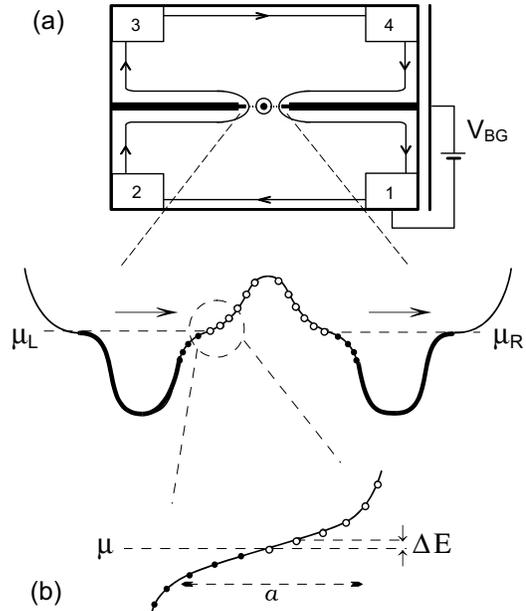

FIG. 2. (a) Illustration of an antidot sample. Numbered rectangles are Ohmic contacts, the antidot is between two front gates. The arrowed lines show an edge channel. The back gate extends over the entire sample on the opposite side of the insulating GaAs substrate. Dotted line in the constriction between the front gates shows tunneling. (b) Self-consistent energy diagram of one Landau level in the constriction. The energy spectrum is continuous at the edges, and discrete near the antidot. (•) and (◦) show filled and empty states, respectively. A blow-up of the QAD-bound states near the chemical potential μ is shown below. At low temperature and excitation $\mu_L - \mu_R$, $k_B T \ll \Delta E$, tunneling occurs between the left and right edges via only one QAD-bound state. The QHE gap forms the tunneling barriers in the QH regime.

When the constriction is on a quantum Hall plateau, particles can tunnel resonantly via the QAD-bound states giving rise to quasiperiodic tunneling conductance G_T peaks, see Fig. 3. [13] A peak in G_T occurs when a QAD-bound state crosses the chemical potential and thus marks the change of QAD occupation by one particle. The experimental fact that the G_T peaks are observed implies that the charge induced in the QAD is quantized, that it comes in discrete particles occupying the antidot-bound states; the G_T peaks mark the change of the population of the QAD by one particle per peak. Measuring G_T as a function of B gives the area of the QAD-bound state through which the tunneling occurs, $S = \phi_0/\Delta B$, where ΔB is the quasiperiod in magnetic field. On the $\nu \approx i$ plateau there are i peaks per ΔB

because i Landau levels are occupied. The above discussion is easy to generalize for $f = \frac{i}{2pi+1}$ FQH plateaus by considering "Landau levels" of composite fermions.

III. QUANTUM ANTIDOT ELECTROMETER

On a QH plateau the charge of quasiparticles localized in the interior of a 2D electron system is well defined. [5,14] In the case of the integer QH plateau at $\nu \approx i$ the quasielectrons are simply electrons in the Landau level $i + 1$, and the quasiholes are the holes in the ith level. It is easy to understand some of the properties of FQHE quasiparticles using composite fermions. [9,10] In the case of the FQH plateau at $f = \frac{i}{2pi+1}$ quasielectrons are composite fermions (an electron binding $2p$ vortices) in the "Landau level" $i + 1$ of composite fermions, and the quasiholes are the holes in the ith level. It has been predicted theoretically that the electric charge of these quasiparticles is $q = \frac{e}{2pi+1}$, positive for the quasiholes and negative for quasielectrons. [8,15] This fascinating fractional quantization of electric charge is a fundamental property of the strongly correlated FQH fluid.
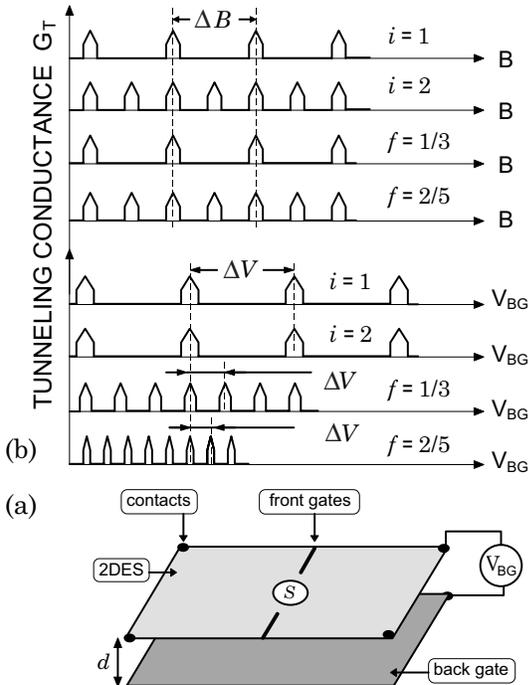

FIG. 3. (a) Schematic of the quantum antidot electrometer. (b) Idealized summary of experimental observations. There is one G_T peak on the $i = 1$ and $f = \frac{1}{3}$ plateaus, and two peaks on the $i = 2$ and $f = \frac{2}{5}$ plateaus observed per period ΔB, with the back gate voltage held constant $V_{BG} = 0$. ΔB gives the size of the QAD: $S = \phi_0/\Delta B$. The same G_T peaks are also observed when a small perpendicular electric field V_{BG}/d is applied to 2DES by biasing the back gate. The period ΔV_{BG} then directly gives the charge q of the QAD-bound particles, in Coulombs, via $q = (\epsilon\epsilon_0\phi_0/d)(\Delta V_{BG}/\Delta B)$.
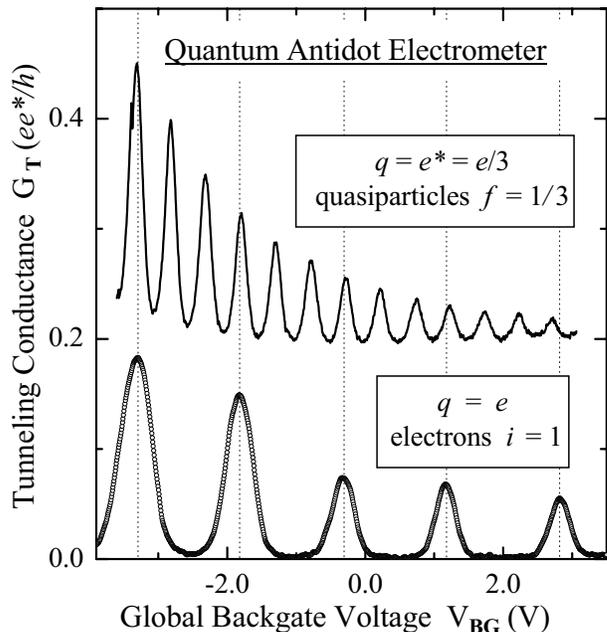

FIG. 4. In the experiment G_T peaks occur each time the occupation of the antidot changes by one particle, an electron for IQHE and a Laughlin quasielectron for the $f = \frac{1}{3}$ FQHE. The measured charge q of the particle is directly proportional to V_{BG}: it takes the same electric field V_{BG}/d to attract three $e/3$ quasiparticles as one electron.

First, we neglect the small ($\sim 10\%$) change of the QAD area S upon change of B. The charge of the QAD-bound particles is then determined directly from the separation of the same G_T peaks as a function of the back gate voltage V_{BG}, Fig. 4. The back gate produces uniform electric field $E_\perp = V_{BG}/d$ which induces a small change of $\epsilon\epsilon_0 E_\perp$ in the 2DES charge density. [16] Classical electrostatics states that the charge induced in the QAD is exactly equal to the 2D charge density induced far from the QAD times the area of the QAD, $\epsilon\epsilon_0 E_\perp/S$. [17] Thus, the charge of one particle q is directly given by the electric field needed to attract one more particle in the area S: $q = \epsilon\epsilon_0 S\Delta V_{BG}/d$, where ΔV_{BG} is the change of the global gate voltage between two consecutive conductance peaks. [18] An absolute and more accurate determination of q uses direct measurement of area $S = \phi_0/\Delta B$ for each QH plateau, as described in Sect. V.

IV. RESONANT TUNNELING EXPERIMENTS

We use low disorder GaAs heterojunction material where 2DES (density 1×10^{11} cm^{-2} and mobility 2×10^6 cm$^2/Vs$) is prepared by exposure to red light at 4.2 K. The antidot-in-a-constriction geometry (somewhat different from that of Refs. [11,12]) was defined by electron

beam lithography on a pre-etched mesa with Ohmic contacts. After ≈ 150 nm chemical etching Au/Ti gate metalization was deposited in the etched trenches. Samples were mounted on sapphire substrates with In metal which serves as the global back gate. All data presented in this paper were taken at 12 mK bath temperature. Extensive cold filtering cuts the electromagnetic background incident on the sample to 5×10^{-17} W, which allows us to achieve a record low effective electron temperature of 18 mK reported for a mesoscopic sample. [19,20]

Figure 5 shows the directly measured four-terminal R_{xx} vs. B data in the integer quantum Hall regime. We use ν_C to denote the filling factor in the constriction region between the front gates, and ν_B for the rest of the sample, "the bulk", away from the constriction region. The front gates are biased negatively in order to bring the edges closer to the antidot to increase the amplitude of the tunneling peaks to a measurable level. This results in ν_C being smaller than ν_B in the bulk. A QHE sample with two ν_B regions separated by a lower ν_C region, if no tunneling occurs, has $R_{xx} = R_L \approx R_{xy}(\nu_C) - R_{xy}(\nu_B)$. The equality is exact, $R_L = \frac{h}{e^2}(1/i_C - 1/i_B)$, if both filling factors ν and ν_B are on a plateau, $\nu_C \approx i_C$ and $\nu_B \approx i_B$, which can be achieved by adjusting the front gate voltages. In such situation the Hall resistances of all regions aquire quantized values. Thus, several R_L plateaus (neglecting tunneling peaks) are seen in Fig. 5.
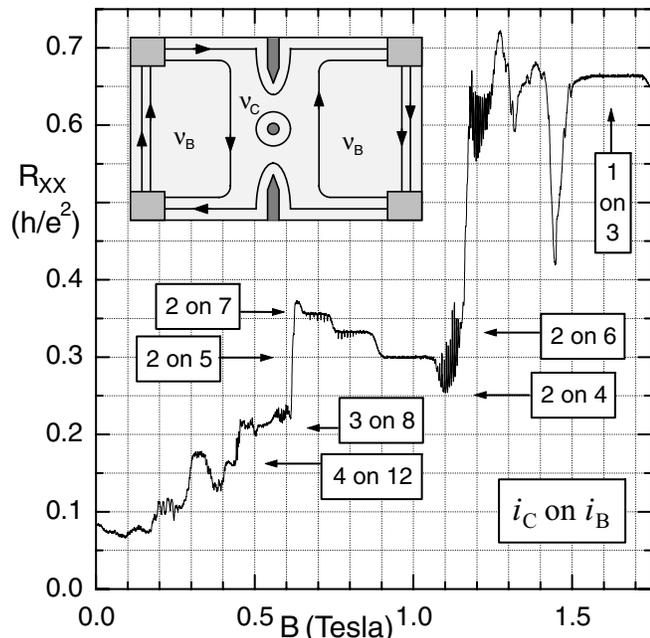

FIG. 5. Four terminal magnetoresistance $R_{(2-3;1-4)} = R_{xx}$ vs. magnetic field B in the integer QH regime. The inset illustrates the sample geometry. The negative bias on the front gates depletes the antidot-constriction region so that ν_C is smaller than ν_B in the bulk. The arrows show quantized values of R_L as discussed in the text.

In some data (cf. Fig. 5) we observe both R_{xx} peaks for $\nu_C < i_C$ ("back scattering") and dips for $\nu_C > i_C$ ("forward scattering"). [13,21] Details of this behavior will be presented elsewhere. [22] The tunneling peaks are superimposed on the smooth R_L background, and we calculate tunneling conductance G_T as described previously. In essence, when R_L is quantized, the edge network model calculation gives [11,19]

$$G_T = \frac{R_{xx} - R_L}{R_H^2 - R_H(R_{xx} - R_L)} \ , \qquad (2)$$

where constriction Hall resistance $R_H = h/i_C e^2$. In the weak tunneling limit $G_T << \frac{e^2}{h}$ this expression reduces to $G_T = (R_{xx} - R_L)/R_H^2$, thus G_T line shape is directly proportional to the directly measured resistance peaks $R_{xx} - R_L$ for weak tunneling.

For the QAD sample presented in this work, from the spacing between the tunneling peaks ΔB we immediately obtain the antidot radius: $r = \sqrt{\phi_0/\pi \Delta B} \approx 345$ nm, the quantum number of the state at the chemical potential $m = \Delta B/B \approx 60, 160,$ and 770 for the centers of the $\nu_C = 2, 1,$ and $1/3$ plateaus, respectively (see Figs. 6 - 8). From the temperature dependence of G_T peak width [22,23] we obtain energy spacing $\Delta E \approx 100 \mu$eV.

V. INVARIANCE OF QUASIPARTICLE CHARGE

FQH quasiparticles are the elementary charged excitations of the FQH condensate, a distinct state of matter that is necessarily separated from all noninteracting electron states by a phase transition. Therefore, FQH quasiparticles are composite objects vastly different from electrons, and are *not* adiabatic images of electrons as are quasiparticle excitations of metals and band insulators (such as IQHE states).

Although six years have passed since the first direct observation of $\frac{1}{3}e$ and $\frac{1}{5}e$ particles in quantum antidot electrometer experiments, [11,12] one crucial aspect of theory remained untested: the invariance of charge at ν far from exact filling. In this section we review recent experiments performed on the integer $i = 1, 2$ and fractional $f = \frac{1}{3}$ quantum Hall plateaus which extend over a filling factor ν range of 27% to 45%. [24] The charge of the QAD-bound quasiparticles has been measured to be constant, independent of ν over the entire plateau extent, with relative accuracy of $\pm 1.2\%$ and absolute accuracy of 3.5%. In addition, we observe no variation of the quasiparticle charge upon variation of temperature or applied current, in the experimentally accessible range.

Figures 6 - 8 show the directly measured four-terminal R_{xx} vs. B data for three QAD plateaus: $i = 2$, $i = 1$ and $f = \frac{1}{3}$. In this Section ν denotes the filling factor in the constriction region. Thus, several R_L plateaus (neglecting tunneling peaks) are seen in Figs. 6 - 8.
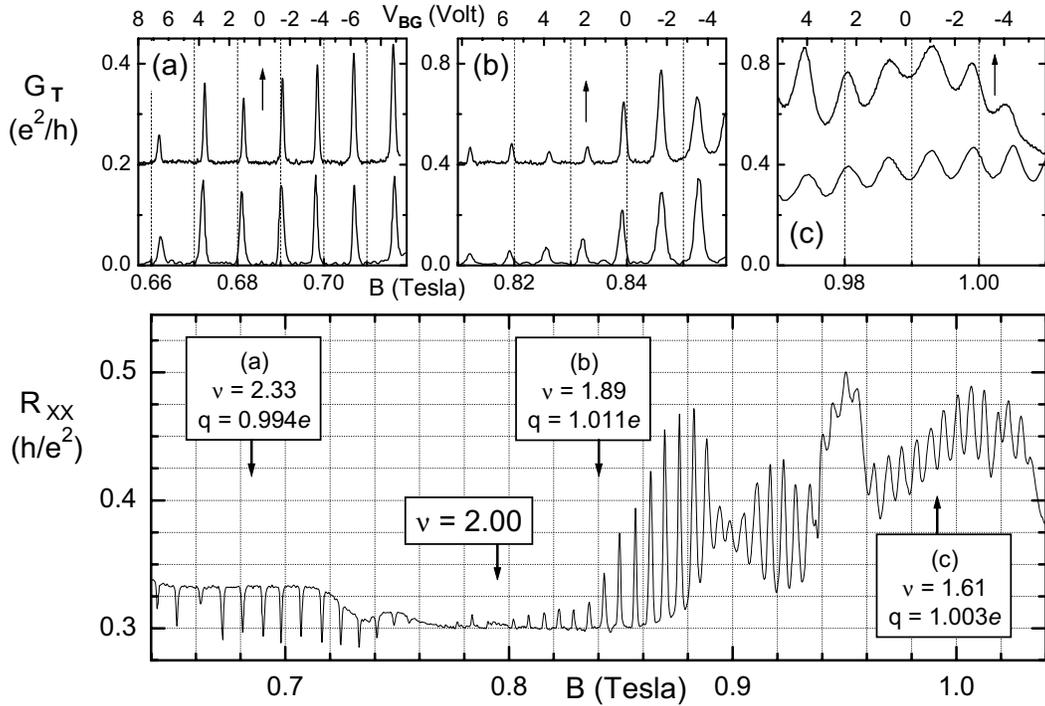
FIG. 6. R_{xx} vs. magnetic field B for the antidot filling ν on the $i = 2$ plateau. The upper panels (a) - (c) give the tunneling conductance measured both as a function of B (back gate voltage is held constant $V_{BG} = 0$), and as a function of V_{BG} (B is held constant, shown by arrows in the lower panel). The G_T vs. V_{BG} curves in panels (a) - (c) are offset vertically.

The tunneling peaks are superimposed on the smooth R_L background, and we calculate G_T as described in Section IV. [11,19] In some data (Figs. 6 and 8) we observe both R_{xx} peaks for $\nu < i$ ("back scattering") and dips for $\nu > i$ ("forward scattering"). [13,21] At several ν on each plateau we took high resolution B-sweeps at $V_{BG} = 0$
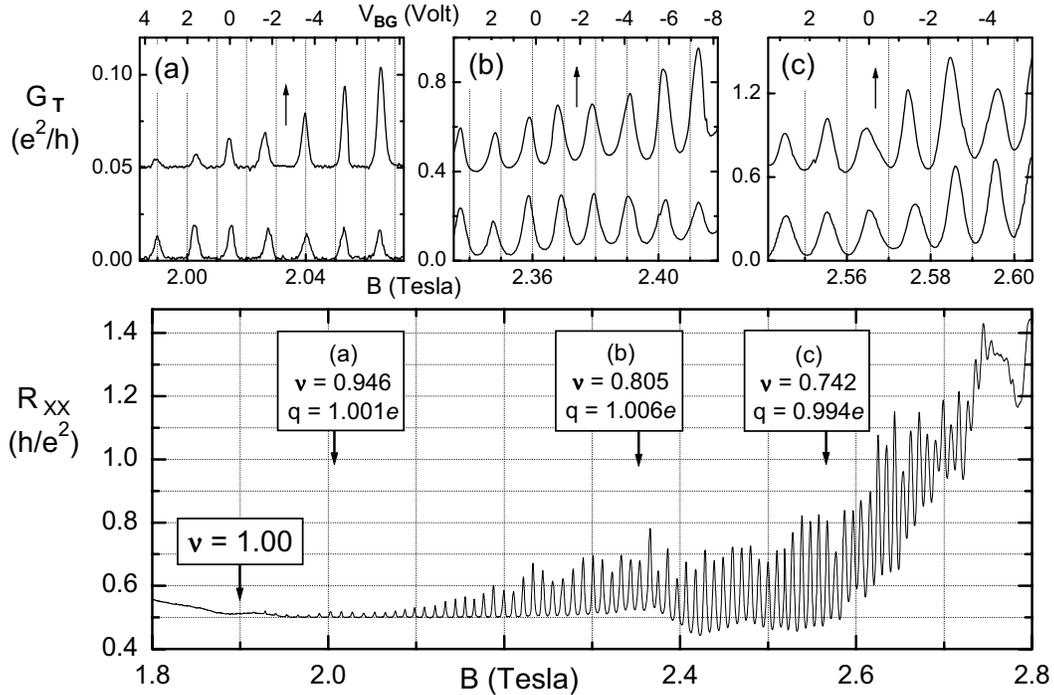

FIG. 7. R_{xx} vs. magnetic field B for the antidot filling ν on the $i = 1$ plateau. The upper panels (a) - (c) give the tunneling conductance measured both as a function of B (back gate voltage $V_{BG} = 0$), and as a function of V_{BG} (B is held constant).
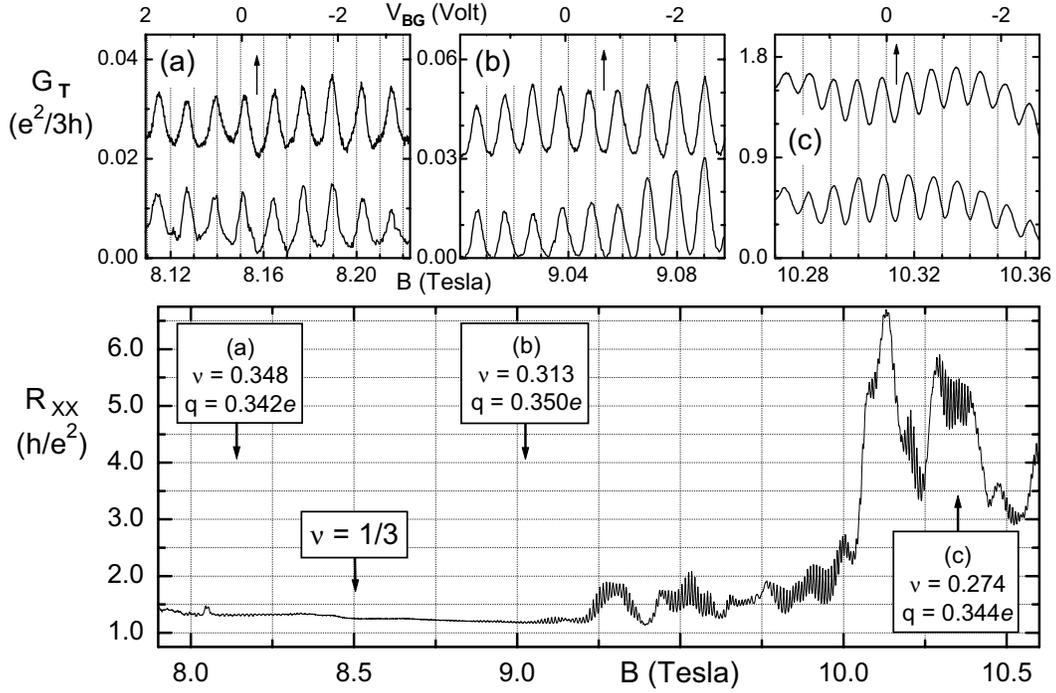
FIG. 8. R_{xx} vs. magnetic field B for the antidot filling ν on the $f = \frac{1}{3}$ plateau. The upper panels (a) - (c) give the tunneling conductance measured both as a function of B (back gate voltage $V_{BG} = 0$), and as a function of V_{BG} (B is held constant).

and, having put the superconducting magnet in the persistent current mode to fix B, we took corresponding sweeps of V_{BG}. Representative G_T vs. B and V_{BG} data are shown in the upper panels of Figs. 6 - 8. [25]
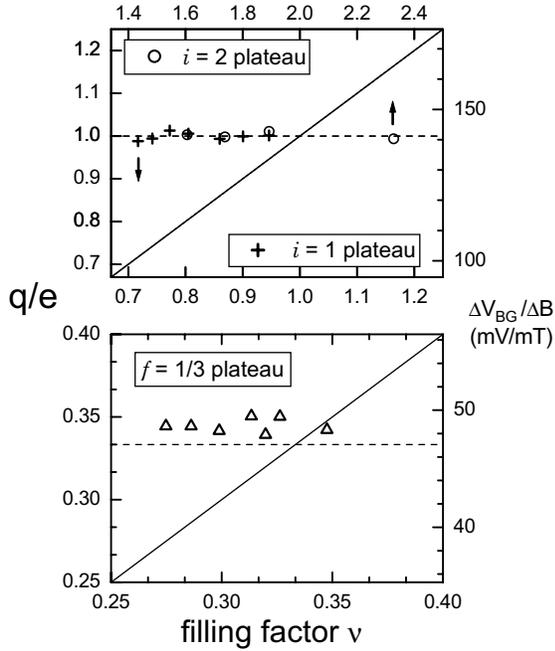

FIG. 9. Summary of the measured QAD-bound particle charge q in units of e. The horizontal dashed lines give the values e and $e/3$. The solid lines have the unit slope. It is quite evident that the ratio $\Delta V_{BG}/\Delta B$ (right axis) is not proportional to ν.

Note that the negative V_{BG} axis direction corresponds to the increasing B. This is so because increasing B results in relative depopulation of QAD independent of the sign of the charge of particles: ψ_m move closer to the center of the QAD, that is, more states move above the chemical potential and become unoccupied. Negative V_{BG} also depopulates the QAD if QAD-bound particles have *negative* charge. Thus the QAD electrometer measures not only the magnitude of q, but also its sign.

The magnitude of the charge q of the QAD-bound particles is then determined from

$$q = \frac{\epsilon\epsilon_0\phi_0}{d} \cdot \frac{\Delta V_{BG}}{\Delta B} \quad \text{in Coulombs,} \tag{3}$$

using the low-temperature GaAs dielectric constant $\epsilon = 12.74$ [26] and the measured thickness of the sample $d \approx 0.430$ mm. The average of the eleven values obtained for the $i = 1, 2$ plateaus $\langle q \rangle_{\text{integer}} = 0.9651e$ is off by 3.5%, more than the standard deviation of $0.0070e$ for the combined data (similar to the results from another electrometer device reported in Refs. [11,12]). We then *normalize* values of q by setting $\langle q \rangle_{\text{integer}} = e$. Thus determined q are given for (a) - (c) data in Figs. 6 - 8, and summarized in Fig. 9. The striking feature of the data of Fig. 9 is that the values of q are constant to a relative accuracy of at least $\pm 1.2\%$ throughout the plateau regions where it was possible to measure the particle charge. The range of ν is about 45% for the $i = 2$ plateau, 27% for the $i = 1$ plateau (the combined normalized ν/i range is 57%), and also 27% for the $f = \frac{1}{3}$ plateau.

We also note that the tunneling current $I_t \approx I G_T / \sigma_{xy}$ is proportional to G_T and thus varies much for peaks of different amplitude, see Fig. 8. Here I is the applied current used to measure R_{xx}; for example, the $f = \frac{1}{3}$ plateau data of Fig. 8 was taken with $I = 50$ pA, and we have measured $\Delta V_{BG}/\Delta B$ with I up to 1 nA, which, combined with the variation in G_T, gives the range of 5×10^{-13} A $\leq I_t \leq 2 \times 10^{-10}$ A. Furthermore, $\Delta V_{BG}/\Delta B$ has been measured in the temperature range 12 mK $\leq T \leq$ 70 mK. [19,20] Under these conditions we observe no change in the value of $q = e/3$ within our experimental accuracy of a few percent. [27] Thus we conclude, based on our experimental results, that the charge of the QH quasiparticles is indeed a well defined quantum number characterizing a particular QHE state.

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